Chapter 4: Bivariate Frequency Distribution & Chi Square Statistic

EXERCISE 4.1 [PAGE 49]

Exercise 4.1 | Q 1. (i) | Page 49

Following tale gives income (X) and expenditure (Y) of 25 families:

Y/X	200 – 300	300 – 400	400 – 500
200 – 300	𝍸 I	𝍸 I	I
300 – 400	–	IIII	𝍸 I
400 – 500	–	–	II

Find Marginal frequency distributions of income and expenditure.

SOLUTION

The bivariate frequency distribution is

Y/X	200 – 300	300 – 400	400 – 500	Total (f_y)
200 – 300	6	6	1	13
300 – 400	0	4	6	10
400 – 500	0	0	2	2
Total (f_x)	6	10	9	25

The Marginal frequency distributions of income (X):

X	200 – 300	300 – 400	400 – 500	Total
Frequency	6	10	9	25

The Marginal frequency distributions of expenditure (Y):

Y	200 – 300	300 – 400	400 – 500	Total

Frequency	13	10	2	25

Exercise 4.1 | Q 1. (ii) | Page 49

Following tale gives income (X) and expenditure (Y) of 25 families:

Y/X	200 – 300	300 – 400	400 – 500
200 – 300	𝍸 I	𝍸 I	I
300 – 400	–	IIII	𝍸 I
400 – 500	–	–	II

Find Conditional frequency distribution of X when Y is between 300 – 400.

SOLUTION

The bivariate frequency distribution is

Y/X	200 – 300	300 – 400	400 – 500	Total (f_y)
200 – 300	6	6	1	13
300 – 400	0	4	6	10
400 – 500	0	0	2	2
Total (f_x)	6	10	9	25

Conditional frequency distribution of X when Y 13 between 300 – 400:

X	200 – 300	300 – 400	400 – 500	Total
Frequency	0	4	6	10

Exercise 4.1 | Q 1. (iii) | Page 49

Following tale gives income (X) and expenditure (Y) of 25 families:

Y/X	200 – 300	300 – 400	400 – 500
200 – 300	𝍸 I	𝍸 I	I
300 – 400	–	IIII	𝍸 I
400 – 500	–	–	II

Find Conditional frequency distribution of Y when X is between 200 – 300.

SOLUTION

The bivariate frequency distribution is

Y/X	200 – 300	300 – 400	400 – 500	Total (f_y)
200 – 300	6	6	1	13
300 – 400	0	4	6	10
400 – 500	0	0	2	2
Total (f_x)	6	10	9	25

Conditional frequency distribution of Y when X is between 200 – 300:

Y	200 – 300	300 – 400	400 – 500	Total
Frequency	6	0	0	6

Exercise 4.1 | Q 1. (iv) | Page 49

Following tale gives income (X) and expenditure (Y) of 25 families:

Y/X	200 – 300	300 – 400	400 – 500
200 – 300	𝍸 I	𝍸 I	I
300 – 400	–	IIII	𝍸 I
400 – 500	–	–	II

Find How many families have their income Rs. 300 and more and expenses Rs. 400 and less?

SOLUTION

Bivariate frequency distribution table for Income (X) and Expenditure (Y) is as follows:

Y/X	200 – 300	300 – 400	400 – 500	Total (f_y)
200 – 300	6	6	1	13
300 – 400	0	4	6	10
400 – 500	0	0	2	2
Total (f_x)	6	10	9	25

The cells 300 – 400 and 400 – 500 are having income ₹ 300 and more and the cells 200 – 300 and 300 – 400 are having expenditure ₹ 400 and less. Now, the following table indicates the number of families satisfying the above condition.

Y/X	300 – 400	400 – 500	Total
200 – 300	6	1	13
300 – 400	4	6	10
Total	10	7	17

∴ There are 17 families with income ₹ 300 and more and expenditure ₹ 400 and less.

Exercise 4.1 | Q 2 | Page 49

Two dice are thrown simultaneously 25 times. The following price of observation are obtained.

(2, 3) (2, 5) (5, 5) (4, 5) (6, 4) (3, 2) (5, 2) (4, 1) (2, 5) (6, 1) (3, 1) (3, 3) (4, 3) (4, 5) (2, 5) (3, 4) (2, 5) (3, 4) (2, 5) (4, 3) (5, 2) (4, 5) (4, 3) (2, 3) (4, 1)

Prepare a bivariate frequency distribution table for the above data. Also, obtain the marginal distributions.

SOLUTION

Let X = Observation on 1st die
Y = Observation on 2nd die

Now, minimum value of X is 1 and maximum value is 6.
Also, minimum value of Y is 1 and maximum value is 6.

Bivariate frequency distribution can be prepared by taking X as row and Y as column.
Bivariate frequency distribution is as follows:

Y/X	1	2	3	4	5	6	Total (f_y)
1	–	–	I	II	–	I	4
2	–	–	I	–	II	–	3
3	–	II	I	III	–	–	6
4	–	–	II	–	–	I	3
5	–	𝍸	–	III	I	–	9
6	–	–	–	–	–	–	0
Total (f_x)	0	7	5	8	3	2	25

Marginal frequency distribution of X:

X	1	2	3	4	5	6	Total
Frequency	0	7	5	8	3	2	25

Marginal frequency distribution of Y:

Y	1	2	3	4	5	6	Total
Frequency	4	3	6	3	9	0	25

Exercise 4.1 | Q 3 | Page 49

Following data gives the age of husbands (X) and age of wives (Y) in years. Construct a bivariate frequency distribution table and find the marginal distributions.

X	27	25	28	26	29	27	28	26	25	25	27
Y	21	20	20	21	23	22	20	20	19	19	23
X	26	29	25	27	26	25	28	25	27	26	
Y	19	23	23	22	21	20	22	23	22	21	

Find conditional frequency distribution of age of husbands when the age of Wife is 23 years.

SOLUTION 1

Let X: Age of husbands in years
Y: Age of Wives in years

Y/X	25	26	27	28	29
1	II	I	–	–	–
2	II	I	–	II	–
3	–	III	I	–	–
4	–	–	III	I	–
5	II	–	I	–	II

Bivariate frequency distribution is

Y/X	2	3	4	5	6	Total ()
1	–	1	2	–	1	4
2	–	1	–	2	–	3
3	2	1	3	–	–	6
4	–	2	–	–	1	3
5	5	–	3	1	–	9
Total ()	7	5	8	3	2	26

Marginal frequency distribution of X (Age of husbands)

X	25	26	27	28	29	Total
()	6	5	5	3	2	21

Margin frequency distribution of Y (Age of wives)

Y	19	20	21	22	23	Total
()	3	5	4	4	5	21

Conditional distribution of X age of husbands) when Y (age of wives) is 23 years.

X	25	26	27	28	29	Total
Frequency	2	–	1	–	2	5

SOLUTION 2

Given, X = Age of Husbands (in years)

Y = Age of Wives (in years)
Now, minimum value of X is 25 and maximum value is 29.

Also, minimum value of Y is 19 and maximum value is 23.
Bivariate frequency distribution is as follows:

Y/X	25	26	27	28	29	Total (f_y)
19	II	I	–	–	–	3
20	II	I	–	II	–	5
21	–	III	I	–	–	4
22	–	–	III	I	–	4
23	II	–	I	–	II	5
Total (f_x)	6	5	5	3	2	21

Marginal frequency distribution of X:

X	25	26	27	28	29	Total
Frequency	6	5	5	3	2	21

Marginal frequency distribution of Y:

Y	19	20	21	22	23	Total
Frequency	3	5	4	4	5	21

Conditional frequency distribution of X when Y is 23:

X	25	26	27	28	29	Total
Frequency	2	0	1	0	2	5

Exercise 4.1 | Q 4 | Page 49

Construct a bivariate frequency distribution table of the marks obtained by students in English (X) and Statistics (Y).

Marks in Statistics (X)	37	20	46	28	35	26	41	48	32	23	20	39	47	33	27	26
Marks in English (Y)	30	32	41	33	29	43	30	21	44	38	47	24	32	21	20	21

Construct a bivariate frequency distribution table for the above data by taking class intervals 20 – 30, 30 – 40, ... etc. for both X and Y. Also find the marginal distributions and conditional frequency distribution of Y when X lies between 30 – 40.

SOLUTION

Let X: Marks in Statistics
Y: Marks in English

Bivariate frequency table can be prepared by taking class intervals 20 – 30, 30 – 40,…, etc for both X and Y. Bivariate frequency distribution is as follows:

Y/X	20 – 30	30 – 40	40 –50	Total (f_y)
20 – 30	II	II	I	5
30 – 40	III	II	II	7
40 – 50	II	I	I	4
Total (f_x)	7	5	4	16

Marginal frequency distribution of X:

X	20 – 30	30 – 40	40 – 50	Total
Frequency	7	5	4	16

Marginal frequency distribution of Y:

Y	20 – 30	30 – 40	40 – 50	Total
Frequency	5	7	4	16

Conditional frequency distribution of Y when X lies between 30 – 40:

Y	20 – 30	30 – 40	40 –50	Total
Frequency	2	2	1	5

Exercise 4.1 | Q 5. (i) | Page 49

Following data gives height in cm (X) and weight in kgs (Y) of 20 boys. Prepare a bivariate frequency table taking class intervals 150-154, 155-159 ...etc. for X and 35-39, 40-44 ...etc. for Y. Also find Marginal frequency distributions.

SOLUTION

Let X: height in cms.
Y: weight in kgs.

Y/X	150 – 154	155 – 159	160 – 164	165 – 169
35 – 39	III	I	–	–
40 – 44	II	III	–	–
45 – 49	I	II	I	–
50 – 54	I	I	III	II

Bivariate frequency distribution is

Y/X	150 – 154	155 – 159	160 – 164	165 – 169	Total (f_y)
35 – 39	3	1	–	–	4
40 – 44	2	3	–	–	5
45 – 49	1	2	1	–	4
50 – 54	1	1	3	2	7
Total (f_x)	7	7	4	2	20

Marginal distribution for X (height in cms)

X	150 – 154	155 – 159	160 – 164	165 – 169	Total

Frequency	7	7	4	2	20

Marginal distribution for Y (weight in kgs)

Y	35 – 39	40 – 44	45 – 49	50 – 54	Total
Frequency	4	5	4	7	20

Exercise 4.1 | Q 5. (ii) | Page 49

Following data gives height in cm (X) and weight in kgs (Y) of 20 boys. Prepare a bivariate frequency table taking class intervals 150-154, 155-159...etc. for X and 35-39, 40-44 ...etc. for Y. Also find conditional frequency distribution of Y when 155 ≤ X ≤ 159
(152, 40) (160, 54) (163, 52) (150, 35) (154, 36) (160, 49) (166, 54) (157, 38) (159, 43) (153, 48) (152, 41) (158, 51) (155, 44) (156, 47) (156, 43) (166, 53) (160, 50) (151, 39) (153, 50) (158, 46)

SOLUTION

Let X: height in cms.
Y: weight in kgs.

Y/X	150 – 154	155 – 159	160 – 164	165 – 169
35 – 39	III	I	–	–
40 – 44	II	III	–	–
45 – 49	I	II	I	–
50 – 54	I	I	III	II

Bivariate frequency distribution is

Y/X	150 – 154	155 – 159	160 – 164	165 – 169	Total (f_y)
35 – 39	3	1	–	–	4
40 – 44	2	3	–	–	5
45 – 49	1	2	1	–	4
50 – 54	1	1	3	2	7
Total (f_x)	7	7	4	2	20

Conditional distribution of Y when 155 ≤ X ≤ 159

Y	35 – 39	40 – 44	45 – 49	50 – 54	Total
Frequency	1	3	2	1	7

EXERCISE 4.2 [PAGES 52 - 53]

Exercise 4.2 | Q 1 | Page 52

Following table shows the classification of applications for secretarial and for sales positions according to gender. Calculate the value of χ^2 Statistic.

	Offered	Denied
Male	75	150
Female	25	50

SOLUTION

Table of observed frequencies.

	Offered	Denied	Row total (R_i)
Male	75	150	225
Female	25	50	75
Column total (C_j)	100	200	300

Expected frequencies are given by

$$E_{ij} = \frac{R_i \times C_j}{N}$$

$$E_{11} = \frac{225 \times 100}{300} = 75$$

$$E_{12} = \frac{225 \times 200}{300} = 150$$

$$E_{21} = \frac{75 \times 100}{300} = 25$$

$$E_{22} = \frac{75 \times 200}{300} = 50$$

Table of expected frequencies.

	Offered	Denied	Total
Male	75	150	225
Female	25	50	75
Total	100	200	300

Now,

$$\chi^2 = \sum\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(75-75)^2}{75} + \frac{(150-150)^2}{150} + \frac{(25-25)^2}{25} + \frac{(50-50)^2}{50}$$

$$= 0$$

Exercise 4.2 | Q 2 | Page 53

200 teenagers were asked which take-out food do they prefer - French fries, burger, or pizza. The results were -

	French fries	Burger	Pizza
Boys	6	20	24
Girls	18	40	92

Compute χ^2 Statistics.

SOLUTION

Table of observed frequencies.

	French fries	Burger	Pizza	Row total (R_i)
Boys	6	20	24	50
Girls	18	40	92	150
Column total (C_j)	24	60	116	200

Expected frequencies are given by

$$E_{ij} = \frac{R_i \times C_j}{N}$$

$$E_{11} = \frac{50 \times 24}{200} = 6$$

$$E_{12} = \frac{50 \times 60}{200} = 15$$

$$E_{13} = \frac{50 \times 116}{200} = 29$$

$$E_{21} = \frac{150 \times 24}{200} = 18$$

$$E_{22} = \frac{150 \times 60}{200} = 45$$

$$E_{23} = \frac{150 \times 116}{200} = 87$$

Table of expected frequencies.

	French fries	Burger	Pizza	Total
Boys	6	15	29	50
Girls	18	45	87	150
Total	24	60	116	200

Now,

$$\chi^2 = \sum \left[\frac{(O_{ij} - E_{ij})^2}{E_{il}} \right]$$

$$= \frac{(6-6)^2}{6} + \frac{(20-15)^2}{15} + \frac{(24-29)^2}{29} + \frac{(18-18)^2}{18} + \frac{(40-45)^2}{45} + \frac{(92-87)^2}{87}$$

$$= 0 + \frac{25}{15} + \frac{25}{29} + 0 + \frac{25}{45} + \frac{25}{87}$$

= 1.67 + 0.86 + 0.56 + 0.29

= 3.38

Exercise 4.2 | Q 3 | Page 53

A sample of men and women who had passed their driving test either in 1st attempt or in 2nd attempt surveyed. Compute χ^2 statistics.

Passed in →	First attempt	Second attempt
Men	32	28
Women	8	12

SOLUTION

Table of observed frequencies.

Passed in →	First attempt	Second attempt	Row total (R_i)
Men	32	28	60
Women	8	12	20
Column total (C_j)	40	40	80

Expected frequencies are given by

$$E_{ij} = \frac{R_i \times C_j}{N}$$

$$E_{11} = \frac{60 \times 40}{80} = 30$$

$$E_{12} = \frac{60 \times 40}{80} = 30$$

$$E_{21} = \frac{20 \times 40}{80} = 10$$

$$E_{22} = \frac{20 \times 40}{80} = 10$$

Table of expected frequencies.

Passed in →	First attempt	Second attempt	Total

Men	30	30	60
Women	10	10	20
Total	40	40	80

Now,

$$\chi^2 = \sum\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(32-30)^2}{30} + \frac{(28-30)^2}{30} + \frac{(8-10)^2}{10} + \frac{(12-10)^2}{10}$$

$$= \frac{4}{30} + \frac{4}{30} + \frac{4}{10} + \frac{4}{10}$$

$$= \frac{8}{30} + \frac{8}{10}$$

$$= 0.27 + 0.8$$

$$= 1.07$$

Exercise 4.2 | Q 4 | Page 53

800 people were asked whether they wear glasses for reading with the following results.

Age	Wear glasses	Do not wear glasses
≤ 30	310	90
> 30	290	110

Compute the χ^2 square statistic.

SOLUTION

Table of observed frequencies.

Age	Wear glasses	Do not wear glasses	Row total (R_i)
≤ 30	310	90	60

> 30	290	110	400
Column total (C_j)	600	200	800

Expected frequencies are given by

$$E_{ij} = \frac{R_i \times C_j}{N}$$

$$E_{11} = \frac{400 \times 600}{800} = 300$$

$$E_{12} = \frac{400 \times 200}{800} = 100$$

$$E_{21} = \frac{400 \times 600}{800} = 300$$

$$E_{22} = \frac{400 \times 200}{800} = 100$$

Table of expected frequencies.

Age	Wear glasses	Do not wear glasses	Total
≤ 30	300	100	400
> 30	300	100	400
Total	600	200	800

Now,

$$\chi^2 = \sum\left[\frac{(o_{ij} - E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(310-300)^2}{300} + \frac{(90-100)^2}{100} + \frac{(290-300)^2}{300} + \frac{(110-100)^2}{100}$$

$$= \frac{100}{300} + \frac{100}{100} + \frac{100}{300} + \frac{100}{100}$$

$$= \frac{200}{300} + 1 + 1$$

$$= 0.67 + 2$$

$$= 2.67$$

Exercise 4.2 | Q 5. (a) | Page 53

Out of a sample of 120 persons in a village, 80 were administered a new drug for preventing influenza and out of them 18 were attacked by influenza. Out of those who were not administered the new drug, 10 persons were not attacked by influenza: Prepare a two-way table showing frequencies.

SOLUTION

The given data can be arranged in the following table.

	Drug administered	**Drug not administered**	**Total**
Attacked	18	-	-
Not Attacked	-	10	-
Total	80	-	120

The observed frequency table can be prepared as follows:

	Drug administered	**Drug not administered**	**Total**
Attacked	18	30	48
Not Attacked	62	10	72
Total	80	40	120

Exercise 4.2 | Q 5. (b) | Page 53

Out of a sample of 120 persons in a village, 80 were administered a new drug for preventing influenza and out of them 18 were attacked by influenza. Out of those who were not administered the new drug, 10 persons were not attacked by influenza: Prepare to compute the χ^2 square statistic.

SOLUTION

The observed frequency table can be prepared as follows:

	Drug administered	Drug not administered	Row total (R_i)
Attacked	18	30	48
Not Attacked	62	10	72
Column total (C_j)	80	40	120

Expected frequencies are given by

$$E_{ij} = \frac{R_{ij} \times C_{ij}}{N}$$

$$E_{11} \frac{48 \times 80}{120} = 32$$

$$E_{12} = \frac{48 \times 40}{120} = 16$$

$$E_{21} = \frac{72 \times 80}{120} = 48$$

$$E_{22} = \frac{72 \times 40}{120} = 24$$

Table of expected frequencies.

	Drug administered	Drug not administered	Total
Attacked	32	16	48
Not Attacked	48	24	72
Total	80	40	120

Now.

$$\chi^2 = \sum\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(18-32)^2}{32} + \frac{(30-16)^2}{16} + \frac{(62-48)^2}{48} + \frac{(10-24)^2}{24}$$

$$= \frac{196}{32} + \frac{196}{16} + \frac{196}{48} + \frac{196}{24}$$

$$= 6.125 + 12.25 + 4.083 + 8.167$$

$$= 30.625$$

MISCELLANEOUS EXERCISE 4 [PAGES 53 - 55]

Miscellaneous Exercise 4 | Q 1. (i) | Page 53

Following data gives the coded price (X) and demand (Y) of a commodity.

Price	5	7	9	8	10	7	9	8	5	11	11	10	2	3	9
Demand	9	15	13	15	14	10	11	14	10	14	6	14	15	11	12
Price	2	4	3	14	6	10	7	15	8	6	5	6	11	14	15
Demand	6	11	8	11	10	15	9	15	13	9	14	10	7	5	6

Classify the data by taking classes 0 – 4, 5 – 9, etc. for X and 5 – 8, 9 – 12, etc. for Y. Also find marginal frequency distribution of X and Y.

SOLUTION

Given,

X = coded price
Y= demand

Bivariate frequency table can be prepared by taking class intervals 0 – 4, 5 – 9,…etc for X and 5 – 8, 9 – 12,… etc for Y.

Bivariate frequency distribution is as follows:

Demand (Y)/Coded price (X)	0 – 4	5 – 9	10 – 14	15 – 19	Total (f_y)
5 – 8	II (2)		III (3)	I (1)	6
9 – 12	II (2)	IIII $\cancel{\text{IIII}}$ (9)	I (1)	–	12
13 – 16	I (1)	$\cancel{\text{IIII}}$ I (6)	IIII (4)	I (1)	12
Total (f_x)	5	15		2	30

Marginal frequency distribution of X:

X	0 – 4	5 – 9	10 – 14	15 – 19	Total
Frequency	5	15	8	2	30

Marginal frequency distribution of Y:

Y	5 – 8	9 – 12	13 – 16	Total
Frequency	6	12	12	30

Miscellaneous Exercise 4 | Q 2. (i) | Page 53

Following data gives the age in years and marks obtained by 30 students in an intelligence test.

Age	16	17	22	19	21	16
Marks	16	19	39	50	48	41
Age	21	20	20	23	22	19
Marks	59	44	42	62	37	67
Age	23	20	22	22	23	22
Marks	45	57	35	37	38	56
Age	17	18	16	21	19	20
Marks	54	61	47	67	49	56
Age	17	18	23	21	20	16
Marks	51	42	65	56	52	48

Prepare a bivariate frequency distribution by taking class intervals 16 – 18, 18 – 20, … etc. for age and 10 – 20, 20 – 30, ... etc. for marks. Find marginal frequency distributions.

SOLUTION

Let
X = Age in years
Y = Marks

Bivariate frequency table can be prepared by taking class intervals 16 – 18, 18 – 20, … etc for X and 10 – 20, 20 – 30, … etc for Y.

Bivariate frequency distribution is as follows:

Marks (Y)/Age in years (X)	**16 – 18**	**18 – 20**	**20 – 22**	**22 – 24**	**Total (f_y)**
10 – 20	II (2)	–	–	–	2
20 – 30	–	–	–	–	0
30 – 40	–	–	–	$\cancel{\text{IIII}}$ (5)	5
40 – 50	III (3)	II (2)	III (3)	I (1)	9
50 – 60	II (2)	I (1)	$\cancel{\text{IIII}}$ (5)	I (1)	9
60 – 70	–	II (2)	I (1)	II (2)	5
Total (f_x)	7	5	9	9	30

Marginal frequency distribution of X:

X	16 – 18	18 – 20	20 – 22	22 – 24	Total
Frequency	7	5	9	9	30

Marginal frequency distribution of Y:

Y	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	Total
Frequency	2	0	5	9	9	5	30

Miscellaneous Exercise 4 | Q 2. (ii) | Page 53

Following data gives the age in years and marks obtained by 30 students in an intelligence test.

Age	16	17	22	19	21	16
Marks	16	19	39	50	48	41
Age	21	20	20	23	22	19
Marks	59	44	42	62	37	67
Age	23	20	22	22	23	22
Marks	45	57	35	37	38	56

Age	17	18	16	21	19	20
Marks	54	61	47	67	49	56
Age	17	18	23	21	20	16
Marks	51	42	65	56	52	48

Prepare a bivariate frequency distribution by taking class intervals 16 – 18, 18 – 20, … etc. for age and 10 – 20, 20 – 30, … etc. for marks. Find conditional frequency distribution of marks obtained when the age of students is between 20 – 22.

SOLUTION

Let
X = Age in years
Y = Marks

Bivariate frequency table can be prepared by taking class intervals 16 – 18, 18 – 20, … etc for X and 10 – 20, 20 – 30, … etc for Y.

Bivariate frequency distribution is as follows:

Marks (Y)/Age in years (X)	**16 – 18**	**18 – 20**	**20 – 22**	**22 – 24**	**Total (f_y)**
10 – 20	II (2)	–	–	–	2
20 – 30	–	–	–	–	0
30 – 40	–	–	–	$\cancel{\text{IIII}}$ (5)	5
40 – 50	III (3)	II (2)	III (3)	I (1)	9
50 – 60	II (2)	I (1)	$\cancel{\text{IIII}}$ (5)	I (1)	9
60 – 70	–	II (2)	I (1)	II (2)	5
Total (f_x)	7	5	9	9	30

Conditional frequency distribution of Y when X is between 20 – 22:

Y	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	Total
Frequency	0	0	0	3	5	1	9

Miscellaneous Exercise 4 | Q 3. (i) | Page 54

Following data gives Sales (in Lakh Rs.) and Advertisement Expenditure (in Thousand Rs.) of 20 firms.

(115, 61) (120, 60) (128, 61) (121, 63) (137, 62) (139, 62) (143, 63) (117, 65) (126, 64) (141, 65) (140, 65) (153, 64) (129, 67) (130, 66) (150, 67) (148, 66) (130, 69) (138, 68) (155, 69) (172, 68)

Construct a bivariate frequency distribution table for the above data by taking classes 115 – 125, 125 –135, ...etc. for sales and 60 – 62, 62 – 64, ...etc. for advertisement expenditure.

SOLUTION

Let X = Sales (in lakh Rs.)
Y = Advertisement Expenditure (in Thousand Rs.)

Bivariate frequency table can be prepared by taking class intervals 115 – 125, 125 – 135, ...etc for X and 60 – 62, 62 – 64, …. etc for Y.

Bivariate frequency distribution is as follows:

Y/X	115 – 125	125 – 135	135 – 145	145 –155	155 – 165	165 – 175	Total (f_y)
60 – 62	II (2)	I (1)	–	–	–	–	3
62 – 64	I (1)	–	III (3)	–	–	–	4
64 – 66	I (1)	I (1)	II (2)	I (1)	–	–	5
66 – 68	–	II (2)	–	II (2)	–	–	4
68 – 70	–	I (1)	I (1)	–	I (1)	I (1)	4
Total (f_x)	4	5	6	3	1	1	20

Miscellaneous Exercise 4 | Q 3. (ii) | Page 54

Following data gives Sales (in Lakh Rs.) and Advertisement Expenditure (in Thousand Rs.) of 20 firms.

(115, 61) (120, 60) (128, 61) (121, 63) (137, 62) (139, 62) (143, 63) (117, 65) (126, 64) (141, 65) (140, 65) (153, 64) (129, 67) (130, 66) (150, 67) (148, 66) (130, 69) (138, 68) (155, 69) (172, 68)

Find marginal frequency distributions

SOLUTION

Let X = Sales (in lakh Rs.)
Y = Advertisement Expenditure (in Thousand Rs.)

Bivariate frequency table can be prepared by taking class intervals 115 – 125, 125 – 135, ...etc for X and 60 – 62, 62 – 64, …. etc for Y.

Bivariate frequency distribution is as follows:

Y/X	**115 – 125**	**125 – 135**	**135 – 145**	**145 – 155**	**155 – 165**	**165 – 175**	**Total (f_y)**
60 – 62	II (2)	I (1)	–	–	–	–	3
62 – 64	I (1)	–	III (3)	–	–	–	4
64 – 66	I (1)	I (1)	II (2)	I (1)	–	–	5
66 – 68	–	II (2)	–	II (2)	–	–	4
68 – 70	–	I (1)	I (1)	–	I (1)	I (1)	4
Total (f_x)	4	5	6	3	1	1	20

Marginal frequency distribution of X:

X	115 – 125	125 – 135	135 – 145	145 – 155	155 – 165	165 – 175	Total
Frequency	4	5	6	3	1	1	20

Marginal frequency distribution of Y:

Y	60 – 62	62 – 64	64 – 66	66 – 68	68 – 70	Total
Frequency	3	4	5	4	4	20

Miscellaneous Exercise 4 | Q 3. (iii) | Page 54

Following data gives Sales (in Lakh ₹) and Advertisement Expenditure (in Thousand ₹) of 20 firms. (115, 61) (120, 60) (128, 61) (121, 63) (137, 62) (139, 62) (143, 63) (117, 65) (126, 64) (141, 65) (140, 65) (153, 64) (129, 67) (130, 66) (150, 67) (148, 66) (130,

69) (138, 68) (155, 69) (172, 68) Conditional frequency distribution of Sales when the advertisement expenditure is between 64 – 66 (Thousand ₹)

SOLUTION

Let X = Sales (in lakh Rs.)
Y = Advertisement Expenditure (in Thousand Rs.)

Bivariate frequency table can be prepared by taking class intervals 115 – 125, 125 – 135, ...etc for X and 60 – 62, 62 – 64, …. etc for Y.

Bivariate frequency distribution is as follows:

Y/X	**115 – 125**	**125 – 135**	**135 – 145**	**145 – 155**	**155 – 165**	**165 – 175**	**Total (f_y)**
60 – 62	II (2)	I (1)	–	–	–	–	3
62 – 64	I (1)	–	III (3)	–	–	–	4
64 – 66	I (1)	I (1)	II (2)	I (1)	–	–	5
66 – 68	–	II (2)	–	II (2)	–	–	4
68 – 70	–	I (1)	I (1)	–	I (1)	I (1)	4
Total (f_x)	4	5	6	3	1	1	20

Conditional frequency distribution of X when Y is between 64 – 66:

X	115 – 125	125 – 135	135 – 145	145 – 155	155 – 165	165 – 175	Total
Frequency	1	1	2	1	0	0	5

Miscellaneous Exercise 4 | Q 3. (iv) | Page 54

Following data gives Sales (in Lakh Rs.) and Advertisement Expenditure (in Thousand Rs.) of 20 firms.
(115, 61) (120, 60) (128, 61) (121, 63) (137, 62) (139, 62) (143, 63) (117, 65) (126, 64) (141, 65) (140, 65) (153, 64) (129, 67) (130, 66) (150, 67) (148, 66) (130, 69) (138, 68) (155, 69) (172, 68)
Conditional frequency distribution of advertisement expenditure when the sales are between 125 – 135 (lakh Rs.)

SOLUTION

Let X = Sales (in lakh Rs.)
Y = Advertisement Expenditure (in Thousand Rs.)

Bivariate frequency table can be prepared by taking class intervals 115 – 125, 125 – 135, ...etc for X and 60 – 62, 62 – 64, …. etc for Y.

Bivariate frequency distribution is as follows:

Y/X	**115 – 125**	**125 – 135**	**135 – 145**	**145 – 155**	**155 – 165**	**165 – 175**	**Total (f_y)**
60 – 62	II (2)	I (1)	–	–	–	–	3
62 – 64	I (1)	–	III (3)	–	–	–	4
64 – 66	I (1)	I (1)	II (2)	I (1)	–	–	5
66 – 68	–	II (2)	–	II (2)	–	–	4
68 – 70	–	I (1)	I (1)	–	I (1)	I (1)	4
Total (f_x)	4	5	6	3	1	1	20

Conditional frequency distribution of Y when X is between 125 – 135:

Y	60 – 62	62 – 64	64 – 66	66 – 68	68 – 70	Total
Frequency	1	0	1	2	1	5

Miscellaneous Exercise 4 | Q 4. (i) | Page 54

Prepare a bivariate frequency distribution for the following data, taking class intervals for X as 35 – 45, 45 – 55, …. etc and for Y as 115 – 130, 130 – 145, … etc. where X denotes the age in years and Y denotes blood pressure for a group of 24 persons.
(55, 151) (36, 140) (72, 160) (38, 124) (65, 148) (46, 130) (58, 152) (50, 149) (38, 115) (42, 145) (41, 163) (47, 161) (69, 159) (60, 161) (58, 131) (57, 136) (43, 141) (52, 164) (59, 161) (44, 128) (35, 118) (62, 142) (67, 157) (70, 162)
Also, find Marginal frequency distribution of X.

SOLUTION

Given X = Age in years
Y = Blood pressure

Bivariate frequency table can be prepared by taking class intervals 35 – 45, 45 – 55, …. etc for X and 115 – 130, 130 – 145, … etc for Y.

Bivariate frequency distribution is as follows:

Blood Pressure (y)/ Age in years(X)	**35 – 45**	**45 – 55**	**55 – 65**	**65 – 75**	**Total (f_y)**
115 – 130	IIII (4)	–	–	–	4
130 – 145	II (2)	I (1)	III (3)	–	6
145 – 160	I (1)	I (1)	II (2)	III 3)	7
160 – 175	I (1)	II (2)	II (2)	II (2)	7
Total (f_x)	8	4	7	5	24

Marginal frequency distribution of X:

X	35 – 45	45 – 55	55 – 65	65 – 75	Total
Frequency	8	4	7	5	24

Miscellaneous Exercise 4 | Q 4. (ii) | Page 54

Prepare a bivariate frequency distribution for the following data, taking class intervals for X as 35 – 45, 45 – 55, …. etc and for Y as 115 – 130, 130 – 145, … etc. where X denotes the age in years and Y denotes blood pressure for a group of 24 persons.

(55, 151) (36, 140) (72, 160) (38, 124) (65, 148) (46, 130) (58, 152) (50, 149) (38, 115) (42, 145) (41, 163) (47, 161) (69, 159) (60, 161) (58, 131) (57, 136) (43, 141) (52, 164) (59, 161) (44, 128) (35, 118) (62, 142) (67, 157) (70, 162)

Also, find Conditional frequency distribution of Y when X ≤ 45.

SOLUTION

Given X = Age in years
Y = Blood pressure

Bivariate frequency table can be prepared by taking class intervals 35 – 45, 45 – 55, …. etc for X and 115 – 130, 130 – 145, … etc for Y.

Bivariate frequency distribution is as follows:

Blood Pressure (y)/ Age in years(X)	**35 – 45**	**45 – 55**	**55 – 65**	**65 – 75**	**Total (f_y)**
115 – 130	IIII (4)	–	–	–	4

130 – 145	II (2)	I (1)	III (3)	–	6
145 – 160	I (1)	I (1)	II (2)	III 3)	7
160 – 175	I (1)	II (2)	II (2)	II (2)	7
Total (f_x)	8	4	7	5	24

Conditional frequency distribution of Y when X < 45:

Y	115 – 130	130 – 145	145 – 160	160 – 175	Total
Frequency	4	2	1	1	8

Miscellaneous Exercise 4 | Q 5 | Page 54

Thirty pairs of values of two variables X and Y are given below. Form a bivariate frequency table. Also, find marginal frequency distributions of X and Y.

X	110	88	91	115	97	85	85	91	120	95
Y	500	800	870	599	625	650	905	700	850	824
X	82	105	99	90	108	124	90	90	111	89
Y	970	609	990	735	600	735	729	840	999	780
X	112	100	87	92	91	82	96	120	121	122
Y	638	850	630	720	695	923	555	810	805	526

SOLUTION

Bivariate frequency table can be prepared by taking class intervals 80 – 90, 90 – 100, …, etc for X and 500 – 600, 600 –700, …., etc for Y.

Bivariate frequency distribution is as follows:

Y/X	**80 – 90**	**90 – 100**	**100 – 110**	**110 – 120**	**120 – 130**	**Total (f_y)**
500 – 600	–	I (1)	–	II (2)	I (1)	4
600 – 700	II (2)	II (2)	II (2)	I (1)	–	7
700 – 800	I (1)	IIII (4)	–	–	I (1)	6
800 – 900	I (1)	III (3)	I (1)	–	III (3)	8
900 – 1000	III (3)	I (1)	–	I (1)	–	5
Total (f_x)	7	11	3	4	5	30

Marginal frequency distribution of X:

X	80 – 90	90 – 100	100 – 110	110 – 120	120 – 130	Total
frequency	7	11	3	4	5	30

Marginal frequency distribution of Y:

Y	500 – 600	600 – 700	700 – 800	800 – 900	900 – 100	Total
Frequency	4	7	6	8	5	30

Miscellaneous Exercise 4 | Q 6 | Page 54

Following table shows how the samples of Mathematics and Economics scores of 25 students are distributed:

Marks in Economics	Marks in Mathematics	
	40 – 70	**70 – 100**
40 – 70	20	15
70 – 100	5	10

Find the value of χ^2 statistic.

SOLUTION

Table of observed frequencies.

Marks in Economics	Marks in Mathematics		Row total (R_i)
	40 – 70	**70 – 100**	
40 – 70	20	15	35
70 – 100	5	10	15
Column total (C_j)	25	25	50

Expected frequencies are given by

$$E_{ij} = \frac{R_i \times C_j}{N}$$

$$E_{11} = \frac{35 \times 25}{50} = 17.5$$

$$E_{12} = \frac{35 \times 25}{50} = 17.5$$

$$E_{21} = \frac{15 \times 25}{50} = 7.5$$

$$E_{22} = \frac{15 \times 25}{50} = 7.5$$

Table of expected frequencies.

Marks in Economics	Marks in Mathematics		Total
	40 – 70	70 – 100	
40 – 70	17.5	17.5	35
70 – 100	7.5	7.5	15
Total	25	25	50

Now,

$$\chi^2 = \sum \left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}} \right]$$

$$= \frac{(20 - 17.5)^2}{17.5} + \frac{(15 - 17.5)^2}{17.5} + \frac{(5 - 7.5)^2}{7.5} + \frac{(10 - 7.5)^2}{7.5}$$

$$= \frac{6.25}{17.5} + \frac{6.25}{17.5} + \frac{6.25}{7.5} + \frac{6.25}{7.5}$$

$$= \frac{12.5}{17.5} + \frac{12.5}{7.5}$$

$= 0.71 + 1.67$

$= 2.38$

Miscellaneous Exercise 4 | Q 7 | Page 54

Compute χ^2 statistic from the following data:

	Graduates	Post-Graduates
Male	28	22
Female	32	18

SOLUTION

Table of observed frequencies.

	Graduates	Post-Graduates	Row total (R_i)
Male	28	22	50
Female	32	18	50
Column total (C_j)	60	40	100

Expected frequencies are given by

$$E_{ij} = \frac{R_i \times C_j}{N}$$

$$E_{11} = \frac{50 \times 60}{100} = 30$$

$$E_{12} = \frac{50 \times 40}{100} = 20$$

$$E_{21} = \frac{50 \times 60}{100} = 30$$

$$E_{22} = \frac{50 \times 40}{100} = 20$$

Table of expected frequencies.

	Graduates	Post-Graduates	Total
Male	30	20	50
Female	30	20	50
Total	60	40	100

Now,

$$\chi^2 = \sum\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(28-30)^2}{30} + \frac{(22-20)^2}{20} + \frac{(32-30)^2}{30} + \frac{(18-20)^2}{20}$$

$$= \frac{4}{30} + \frac{4}{20} + \frac{4}{30} + \frac{4}{20}$$

$$= \frac{8}{30} + \frac{8}{20}$$

$$= 0.267 + 0.4$$

$$= 0.667$$

Miscellaneous Exercise 4 | Q 8 | Page 54

Attitude of 250 employees towards a proposed policy of the company is as observed in the following table. Calculate the χ^2 statistic.

	Favor	Indifferent	Oppose
Male	68	46	36
Female	27	49	24

SOLUTION

Table of observed frequencies

	Favor	Indifferent	Oppose	Row total (R_i)
Male	68	46	36	150

Female	27	49	24	100
Column total (C_j)	95	95	60	250

Expected frequencies are given by

$$E_{ij} = \frac{R_i \times Cj}{N}$$

$$E_{11} = \frac{150 \times 95}{250} = 57$$

$$E_{12} = \frac{150 \times 95}{250} = 57$$

$$E_{13} = \frac{150 \times 60}{250} = 36$$

$$E_{21} = \frac{100 \times 95}{250} = 38$$

$$E_{22} = \frac{100 \times 95}{250} = 38$$

$$E_{23} = \frac{100 \times 60}{250} = 24$$

Table of expected frequencies.

	Favor	**Indifferent**	**Oppose**	**Row total (R_i)**
Male	57	57	36	150
Female	38	38	24	100
Column total (C_j)	95	95	60	250

Now,

$$\chi^2 = \sum\left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right]$$

$$= \frac{(68-57)^2}{57} + \frac{(46-57)^2}{57} + \frac{(36-36)^2}{36} + \frac{(27-38)^2}{38} + \frac{(49-38)^2}{38} + \frac{(24-24)^2}{24}$$

$$= \frac{121}{57} + \frac{121}{57} + 0 + \frac{121}{38} + \frac{121}{38} + 0$$

$$= \frac{242}{57} + \frac{242}{38}$$

$$= 4.246 + 6.368$$

$$= 10.614$$

Miscellaneous Exercise 4 | Q 9 | Page 55

In a certain sample of 1000 families, 450 families are consumers of tea. Out of 600 Hindu families, 286 families consume tea. Calculate the χ^2 statistic.

SOLUTION

The given data can be arranged in the following table.

	Consume Tea	Do not consume Tea	Total
Hindu family	286		600
Non Hindu family			
Total	450		1000

Table of observed frequencies.

	Consume Tea	Do not consume Tea	Row total (R_i)
Hindu family	286	314	600
Non Hindu family	164	236	400
Column total (C_j)	450	550	1000

Expected frequencies are given by

$$E_{ij} = \frac{R_i \times Cj}{N}$$

$$E_{11} = \frac{600 \times 450}{1000} = 270$$

$$E_{12} = \frac{600 \times 550}{1000} = 330$$

$$E_{21} = \frac{400 \times 450}{1000} = 180$$

$$E_{22} = \frac{400 \times 550}{1000} = 220$$

Table of expected frequencies.

	Consume Tea	Do not consume Tea	Total
Hindu family	270	330	600
Non Hindu family	180	220	400
Total	450	550	1000

Now,

$$\chi^2 = \sum \left[\frac{(o_{ij} - E_{ij})^2}{E_{ij}} \right]$$

$$= \frac{(286 - 270)^2}{270} + \frac{(314 - 330)^2}{330} + \frac{(164 - 180)^2}{180} + \frac{(236 - 220)^2}{220}$$

$$= \frac{256}{270} + \frac{256}{330} + \frac{256}{180} + \frac{256}{220}$$

= 0.948 + 0.776 + 1.422 + 1.164

= 4.31

Miscellaneous Exercise 4 | Q 10 | Page 55

A sample of boys and girls were asked to choose their favourite sport, with the following results. Find the value of the χ^2 statistic.

	Football	Cricket	Hockey	Basketball
Boys	86	60	44	10
Girls	40	30	25	5